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Vertex Operators of Open String States in the Intersecting D-brane World

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Starting with a new bosonization scheme for the $\beta\gamma$ CFT of the super-conformal ghosts, vertex operators are constructed for massless open string states in the intersecting D-brane world. These vertex operators satisfy all requirements for a consistent RNS formulation of superstring theories, so GSO projections can be properly taken.

With the advent of D-branes, studies have been intensified for phenomenologies of Type I/II string theories in recent years. D-brane configurations permit chiral fermions and the gauge group structure $U(3) \times U(2) \times U(1)$. Standard-like models and their supersymmetric extensions have been constructed from orbifolds or orentifolds of Type II string theories with D-branes[1, 2], in particular from the intersecting D-brane world[3]. In these models, GSO projections, which are closely related to vertex operators, are crucial to maintain the chiralities of massless states in the Ramond (R) sector as well as the $N = 1$ supersymmetry. However, a proper definition of vertex operators seems to be wanting for open strings connecting D-branes which are intersecting with nontrivial angles[4].

Vertex operator is a concept of 2-dimensional conformal field theories (CFTs). They play an important role in superstring theories and provide elegant (conformally invariant) means to calculate string interactions. Fermion vertex operators, in particular, are needed to define the worldsheet spinor number and GSO projections[5]. They help to expose the space-time supersymmetries in the manifestly Lorentz invariant Ramond-Neveu-Schwarz (RNS) formulation of superstring theories. By construction, vertex operators turn physical states into physical states. Fermionic vertex operators V_F convert fermions into bosons and vice versa, which form spinors of the Lorentz group $SO(1, p)$ $(p \leq 9)$. Most importantly, vertex operators should have a conformal dimension of *one*. Otherwise, vertex operators cannot turn a physical state into another physical state and a covariant quantization cannot be formulated for superstring theories. The realization of these conditions could be challenging, but can be greatly simplified by the "bosonization" technique[6, 7].

In Type I superstring theories with D9-branes, where open strings obey only Neumann-Neumann boundary conditions, the construction of vertex operators was based upon the so-called FMS bosonization scheme of the $\beta\gamma$ CFT of the super-conformal ghosts[5]. The vertex operator of the super-conformal ghost ground state, with conformal dimension 3/8 in the R sector and 1/2 in the NS sector, provided the long-missing pieces to the full vertex operators with the correct conformal dimension. Introducing nontrivial D-branes into Type I/II superstring theories, open strings can also obey Neumann-Dirichlet or Dirichlet-Neumann conditions. A naive generalization of the FMS prescription, however, does not yield qualified vertex operators. The failure can be illustrated by an example of the supersymmetric "$n = 2$" case with D-branes intersecting at angles[4]. The brane configuration consists of two 2-branes in four dimensions $Z_3 = X_6 + iX_7$ and $Z_4 = X_8 + iX_9$. The first 2-brane is oriented along the $X_{6,8}$-axes. The second one is obtained by rotating the first one by an angle $\alpha\pi$ in the Z_3 plane and $-\alpha\pi$ in the Z_4 plane $(0 \leq \alpha \leq \frac{1}{2})$. For massless open strings connecting two such 2-branes, a straightforward generalization of the FMS scheme would result in the following vertex operators

$$
\begin{aligned}
&e^{i(1-\alpha)H_3} e^{i\alpha H_4} e^{-\phi} , \\
&e^{-i\alpha H_3} e^{-i(1-\alpha)H_4} e^{-\phi} ,
\end{aligned}
\tag{1}
$$

in the NS sector[4]. Here the worldsheet fermions $\Psi^\mu(z)$ are bosonized by free holomorphic scalars $H_a(z)$[8] and $e^{-\phi}$ is from the super-conformal ghosts. The vertex operators in Eq.(1) have conformal dimension $h = 1 - \alpha + \alpha^2$, which is not equal to *one* unless α vanishes. Constructions in the R sector yield similar results. A close inspection shows that the mismatch can only be due to the super-conformal ghost part. The factor $e^{-\phi}$ does not reflect the influence of the open string boundary conditions correctly.

In this letter, we provide a new bosonization scheme of the $\beta\gamma$ CFT. Based upon this, we construct a set of new ghost vertex operators, which provide the needed extra pieces to the full vertex operators of open string states with supersymmetries[4]. These full vertex operators satisfy all requirements for a consistent RNS formulation of superstring theories, so that GSO projections can be properly taken.

An Alternative Bosonization of the $\beta\gamma$ CFT. We are interested in the $\beta\gamma$ CFT from the BRST quantization of superstring theories, where $\beta(z)$ and $\gamma(z)$ are Faddeev-Popov super-conformal ghosts from gauge-fixing the superstrings. The classical action of the $\beta\gamma$ CFT is,

$$S = \frac{1}{2\pi}\int d^2z\beta\bar{\partial}\gamma \ . \tag{2}$$

β and γ are commuting fields and could be regarded as "bosons" in 2-dimensional Euclidean space. They have the following operator product expansions (OPEs),

$$\begin{array}{ll} \beta(z)\gamma(0) = -\frac{1}{z} + O(1)\,, & \gamma(z)\beta(0) = \frac{1}{z} + O(1)\,, \\ \beta(z)\beta(0) = O(1)\,, & \gamma(z)\gamma(0) = O(1)\,. \end{array} \tag{3}$$

In general, a conformal invariant system may have more than one conserved energy-momentum tensor. To give the conformal dimensions, $h_\beta = 3/2$ and $h_\gamma = -1/2$, respectively, as required by the BRST quantization procedure, the energy-momentum tensor of the $\beta\gamma$ CFT is uniquely defined as,

$$T_{\beta\gamma}(z) =: [\partial\beta(z)]\gamma(z) : -\frac{3}{2} : \partial[\beta(z)\gamma(z)] :, \tag{4}$$

so the central charge of the theory is $c_{\beta\gamma} = 11$.

Though $\beta(z)$ and $\gamma(z)$ are bosonic, they need to be further "bosonized" in order to conveniently construct the vertex operators of ghost ground states[8]. By bosonization it is meant that these fields are re-expressed as operator exponentials of other boson fields. In the FMS scheme[5], the bosonization was realized in terms of a holomorphic scalar $\phi(z)$ and a pair of anti-commutating bosonic ghosts $\xi(z)$ and $\eta(z)$,

$$\beta(z) \cong \exp[-\phi(z)]\partial\xi(z), \quad \gamma(z) \cong \exp[\phi(z)]\eta(z). \tag{5}$$

These "bosons" form two separate 2-dimensional CFTs. In the ϕ CFT, the fundamental OPE is $\phi(z)\phi(0) \sim -\ln z$ and the energy-momentum tensor is,

$$T_\phi(z) = -\frac{1}{2} : [\partial\phi(z)][\partial\phi(z)] : -\partial^2\phi(z). \tag{6}$$

$\xi(z)$ and $\eta(z)$ are decoupled from ϕ and form an anti-commutating CFT[8]. The fundamental OPEs of the $\xi\eta$ CFT are postulated to be

$$\begin{array}{ll} \xi(z)\eta(0) = \frac{1}{z} + O(1)\,, & \eta(z)\xi(0) = \frac{1}{z} + O(1)\,, \\ \partial\xi(z)\partial\xi(0) = O(z)\,, & \eta(z)\eta(0) = O(z), \end{array} \tag{7}$$

and the energy-momentum tensor is

$$T_{\xi\eta}(z) = -\eta(z)\partial\xi(z). \tag{8}$$

As a result, the "bosonic ghosts" $\eta(z)$ and $\xi(z)$ have conformal dimensions $h_\eta = 1$ and $h_\xi = 0$, respectively. Of course, $T_{\beta\gamma}(z) \cong T_\phi(z) + T_{\xi\eta}(z)$.

To construct fermion vertex operators of the open states in the R sectors, the FMS bosonization of the $\beta\gamma$ CFT is only useful when the full vertex operators form a spinor of the Lorentz group $SO(1,9)$. In Type II superstrings, open string states arise along with the appearance of BPS D-branes. In these theories, open strings obey Neumann boundary conditions in directions parallel to the D-brane's worldvolume while Dirichlet conditions in directions perpendicular to the worldvolume. The open string fermion vertex operators are generally spinors of the Lorentz group $SO(1,p)$ with $p \leq 9$. For $p < 9$, the FMS scheme fails. As mentioned above, the conformal dimension of vertex operators based upon the FMS bosonization does not equal to *one* in general. Technically, the failure is due to the fact that the number of independent free bosons in FMS bosonization is too small and there are no adjustable parameters to satisfy the $h = 1$ requirement.

We now propose an alternative bosonization scheme for the $\beta\gamma$ CFT. Instead of one free holomorphic boson, we introduce four and postulate the following equivalence relations,

$$\begin{aligned} \beta(z) &\cong \exp[-\phi_1(z) - \phi_2(z) - \frac{1}{\sqrt{2}}\chi_1(z) - \frac{1}{\sqrt{2}}\chi_2(z)]\partial\xi(z), \\ \gamma(z) &\cong \exp[\ \phi_1(z) + \phi_2(z) + \frac{1}{\sqrt{2}}\chi_1(z) + \frac{1}{\sqrt{2}}\chi_2(z)]\eta(z). \end{aligned} \tag{9}$$

$\xi(z)$ and $\eta(z)$ are same as those in the FMS bosonization, which obey the OPEs in Eq. (7) and have the energy-momentum tensor in Eq. (8). $\phi_i(z)$ and $\chi_i(z)$ $(i = 1,\, 2)$, which are decoupled from the $\xi\eta$ CFT and independent of each other, have the following OPEs,

$$\begin{array}{ll} \phi_i(z)\phi_j(0) \sim -\ \delta_{ij}\ln z\,, & \\ \chi_i(z)\chi_j(0) \sim \ \ \delta_{ij}\ln z\,, & (i,\, j = 1,\, 2) \end{array} \tag{10}$$

and energy-momentum tensors,

$$\begin{array}{ll} T_{\phi_i}(z) = -\ \frac{1}{2} : [\partial\phi_i(z)][\partial\phi_i(z)] : -\partial^2\phi_i(z), & \\ T_{\chi_i}(z) = \ \ \frac{1}{2} : [\partial\chi_i(z)][\partial\chi_i(z)] : +\omega_i\partial^2\chi_i(z)\,, & (i = 1,\, 2) \end{array} \tag{11}$$

with constants ω_1 and ω_2 to be be determined. Similar to the FMS scheme, the sum of the energy-momenta of the $\xi\eta$, ϕ_i, and χ_i CFTs is equivalent to that of the $\beta\gamma$ CFT,

$$T_{\beta\gamma}(z) \cong \sum_{i=1}^{2}[T_{\phi_i}(z) + T_{\chi_i}(z)] + T_{\xi\eta}(z). \tag{12}$$

Now we determine the constants ω_1 and ω_2. The central charges of the newly introduced CFTs are

$$\begin{array}{ll} c_{\phi_1} = c_{\phi_2} = 13\,, & c_{\chi_1} = 1 - 12\omega_1^2\,, \\ c_{\chi_2} = 1 - 12\omega_2^2\,, & c_{\xi\eta} = -2\,. \end{array} \tag{13}$$

Their sum should reproduce the central charge $c_{\beta\gamma} = 11$. This yields the first constraint on ω_1 and ω_2,

$$\omega_1^2 + \omega_2^2 = \frac{5}{4}. \tag{14}$$

To reproduce the correct conformal dimensions of $\beta(z)$ and $\gamma(z)$, one needs,

$$\omega_1 + \omega_2 = -\sqrt{2}\,. \tag{15}$$

The solution to Eqs.(14) and (15) is unique,

$$\omega_1 = -\frac{1}{4}\sqrt{2}, \quad \omega_2 = -\frac{3}{4}\sqrt{2}. \tag{16}$$

This completes our bosonization proposal.

Super-conformal Ghost Current. For completeness, we re-express the super-conformal ghost current $J_{\beta\gamma}(z) =: \beta(z)\gamma(z):$ in the new bosonization scheme

$$J_{\beta\gamma}(z) \cong \partial\phi_1(z) + \partial\phi_2(z) + \frac{1}{\sqrt{2}}[\partial\chi_1(z) + \partial\chi_2(z)]\,. \tag{17}$$

This can be easily checked by calculating the OPE of $\beta(z)\gamma(-z)$. In terms of the original ghost fields β and γ, one has

$$\beta(z)\gamma(-z) = -\frac{1}{2z} + :\beta(0)\gamma(0): + O(z)\,. \tag{18}$$

In terms of the bosonized fields, one has alternatively,

$$\beta(z)\gamma(-z) \cong -\tfrac{1}{2z} \;+\{\,\partial\phi_1(0) + \partial\phi_2(0) + \tfrac{1}{\sqrt{2}}\,[\,\partial\chi_1(0) + \partial\chi_2(0)\,]\,\} + O(z)\,. \tag{19}$$

The equivalence between Eqs.(18) and (19) ensures the validity of Eq.(17). The super-conformal ghost current is actually equivalent to a special algebraic sum of $\phi - \chi$ momentum currents. For a consistent check, we calculate the following OPEs in terms of the bosonized fields,

$$\begin{aligned} J_{\beta\gamma}(z)\beta(0) &= \frac{1}{z}\beta(0) + O(1), \\ J_{\beta\gamma}(z)\gamma(0) &= -\ \frac{1}{z}\gamma(0) + O(1). \end{aligned} \tag{20}$$

This is just what to be expected, the ghost numbers of β and γ are 1 and -1, respectively.

Vertex Operators for Massless Open String States. Now we are ready to construct the vertex operators for massless open string states in intersecting D-brane scenarios. For concreteness, we consider open strings connecting two 2-branes which were described above Eq.(1). The D-brane configuration preserves part of the space-time supersymmetries in open string sectors. The vertex operators are of the forms

$$\begin{aligned} V_B^{(1)}(z) &= e^{[-i(1-\alpha)H_3(z) - i\alpha H_4(z)]}\Theta_{gh}^{NS}(z) \\ V_B^{(2)}(z) &= e^{[i\alpha H_3(z) + i(1-\alpha)H_4(z)]}\Theta_{gh}^{NS}(z) \end{aligned} \tag{21}$$

in the NS sector[4] and

$$\begin{aligned} V_F(z) = \; & e^{[is_0 H_0(z) + is_1 H_1(z) + is_2 H_2(z)]} \\ & \cdot e^{[i(\alpha-\frac{1}{2})H_3(z) + i(\frac{1}{2}-\alpha)H_4(z)]} \cdot \Theta_{gh}^{R}(z) \end{aligned} \tag{22}$$

in the R sector (where $s_0, s_1, s_2 = \pm\frac{1}{2}$). The superghost factors of the vertex operators obey the OPEs[8],

$$\gamma(z)\Theta_{gh}^{NS}(0) = O(z)\,, \quad \beta(z)\Theta_{gh}^{NS}(0) = O(1/z)\,. \tag{23}$$

$$\gamma(z)\Theta_{gh}^{R}(0) = O(\sqrt{z})\,, \quad \beta(z)\Theta_{gh}^{NS}(0) = O(1/\sqrt{z}) \tag{24}$$

These imply the general forms

$$\begin{aligned} \Theta_{gh}^{NS}(z) &= e^{[-(1-\alpha)\phi_1(z) - \alpha\phi_2(z) + \rho\chi_1(z) - \rho\chi_2(z)]}\,, \\ \Theta_{gh}^{R}\,(z) &= e^{[-(\frac{1}{2}-\alpha)\phi_1(z) - \alpha\phi_2(z) + \kappa\chi_1(z) - \kappa\chi_2(z)]} \end{aligned} \tag{25}$$

for the vertex operators of the ghost ground states. For $\rho = 0,\ 1/\sqrt{2}$ and $\kappa = (1 \pm \sqrt{4\alpha+1})/2\sqrt{2}$, the full vertex operators do have the required conformal dimension of *one*. As expected, the fermion vertex operators $V_F(z)$ in Eq. (22) form a spinor of the Lorentz group $SO(1,5)$. GSO projections can be implemented by imposing mutual locality of the vertex operators[4, 8]. This is automatically satisfied by vertex operators in Eq. (21) in the NS sector. In the R sectors, this requires $s_0 + s_1 + s_2 = 1/2$. Taking into account the physical state condition $s_0 = 1/2$, we have the following vertex operator for left-handed massless open string state in the R sector ($s_1 = -1/2$),

$$\begin{aligned} V_F(z) \;=\; & \exp\{\frac{i}{2}[\,H_0(z) - H_1(z) + H_2(z)\,] + i(\alpha - \frac{1}{2})H_3(z) \\ & + i(\frac{1}{2} - \alpha)H_4(z)\}\cdot\exp[-(\frac{1}{2} - \alpha)\phi_1(z) - \alpha_2\phi_2(z) \\ & + \kappa\chi_1(z) - \kappa\chi_2(z)] \end{aligned} \tag{26}$$

Within the BRST quantization, one can define a nilpotent charge,

$$\begin{aligned} Q \;=\; & \frac{1}{2\pi i}\oint dz\{\,c(z)T_B^M(z) + \gamma(z)T_F^M(z) \\ & + b(z)c(z)[\,\partial c(z)\,] + \frac{3}{4}\,[\,\partial c(z)\,]\,\beta(z)\gamma(z) \\ & + \frac{1}{4}c(z)\,[\,\partial\beta(z)\,]\,\gamma(z) - \frac{3}{4}c(z)\beta(z)\,[\,\partial\gamma(z)\,] \\ & - b(z)\gamma^2(z)\,\}\,. \end{aligned} \tag{27}$$

where b, c are the usual conformal ghosts, T_B^M is the energy-momentum tensor of matter fields and T_F^M its superconformal partner. The physical states form a BRST cohomology class,

$$Q|\mathrm{Phys}>= 0\,, \tag{28}$$

It is straightforward to verify that

$$[\,Q,\, \frac{1}{2\pi i}\oint dz V_{B,F}(z)\,] \;= 0. \tag{29}$$

So, if $|\Phi>$ is a physical state, $\frac{1}{2\pi i}\oint dz V_{F,B}(z)|\Phi>$ is a physical state as well. $V_{B,F}(z)$ defined in this letter do indeed satisfy all conditions as required.

It should pointed out that our construction is quite general and can be extended (at least to cases where there are some unbroken supersymmetries in open string sectors). For illustration, we present the results for the supersymmetric "$n = 3$" case in the sense of [4], where D-branes intersecting with two independent angles α_2 and

α_3. The vertex operators for massless open string states are found to be

$$\begin{aligned} V_B(z) &= \exp\{i[\alpha_2 H_2(z) + \alpha_3 H_3(z) + (1-\alpha_2-\alpha_3)H_4(z)]\} \\ &\quad \cdot \exp[-(1-\alpha_2-\alpha_3)\phi_1(z) - (\alpha_2+\alpha_3)\phi_2(z) \\ &\quad +\rho\chi_1(z) - \rho\chi_2(z)] \end{aligned} \tag{30}$$

in the NS sector and

$$\begin{aligned} V_F(z) &= \exp\{\pm\frac{i}{2}[H_0(z)+H_1(z)] + i(\alpha_2-\frac{1}{2})H_2(z) \\ &\quad +i(\alpha_3-\frac{1}{2})H_3(z) + i(\frac{1}{2}-\alpha_2-\alpha_3)H_4(z)\} \\ &\quad \cdot\exp[-(\frac{1}{2}-\alpha_2-\alpha_3)\phi_1(z) - (\alpha_2+\alpha_3)\phi_2(z) \\ &\quad +\kappa\chi_1(z) - \kappa\chi_2(z)] \end{aligned} \tag{31}$$

in the R sector. The condition $h=1$ for these full vertex operators is satisfied if ρ and κ are taken as

$$\begin{aligned} \rho &= \frac{1}{2\sqrt{2}}\,(\,1 \pm \sqrt{1+8\alpha_2\alpha_3}\,)\,, \\ \kappa &= \frac{1}{2\sqrt{2}}\,[\,1 \pm \sqrt{4(\alpha_2+\alpha_3)+8\alpha_2\alpha_3+1}\,]\,. \end{aligned} \tag{32}$$

These vertex operators commute with the BRST charge. Furthermore, the fermion vertex operators form a spinor of the Lorentz group $SO(1,3)$. GSO projections can be carried out by imposing mutual locality of these vertex operators.

In conclusion, we have constructed a new set of vertex operators for supersymmetric open string states obeying general boundary conditions. The prescription relies on an alternative bosonization of the $\beta\gamma$ CFT, in which four independent holomorphic free bosons have been used, apart from the conventional anti-commutating $\xi\eta$ ghosts. The new bosonization extends to the fundamental OPEs, the energy-momentum tensor, the super-conformal ghost current, and thus all correlation functions. Although the new scheme looks slightly complicated, it is very powerful and provides sufficient rooms for accommodating the requirements to construct reliable vertex operators for open string states. The procedure can be readily extended to non-supersymmetric cases[10].

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